Mutual Synchronization of Closed Kinematic Chains

R. Peón, C. Acosta, O. Palma, C. Villanueva
Facultad de Ingeniería, Universidad Autónoma de Yucatán
Av. Industrias No Contaminantes S/N, Mérida, Yuc., Apdo. 150 Cordemex
Teléfono:(52)999-9-30-05-70

G. Solís-Perales
Depto. de Electrónica, CUCEI, Universidad de Guadalajara
Av. Revolución 1500, Guadalajara, Jal., 44430, México
Teléfono: (+52)33-1378-5900, ext. 7783

***Abstract*— In this contribution we present the mutual synchronization of an ensemble of robots with closed kinematic chain. Parallel kinematic chains possess the advantages of high stiffness, low inertia, and large payload capacity. The main idea is to achieve synchronization in an group of robots composed by planar five-bar parallel robots with two D.O.F.s. The synchronization is achieved into the workspace, i.e., the end position of each robot is equal to the rest of the robots. The mutual synchronization is achieved by means of a torque-computed PD controller and numerical simulations are provided to illustrate the results.**

Keywords: Mutual Synchronization, Parallel Robot Syncronization.

I. Introduction

The definition of synchronization was firstly introduced by Christian Huygens in the 16th century, and means to share the space at the same time (Pikovsky *et al.*, 2008). This definition was presented by Afraimavich applied to dynamical systems (Afraimovich *et al.*, 1986), after that it was applied to complex chaotic systems (Femat and Solís-Perales, 2008), where control techniques have been applied with a variety of results. However, in the field of mechanical systems, the synchronization has been studied as a cooperation or a coordination problem, where a task cannot be carried out by a single system or robot (Nijmeijer and Rodriguez-Angeles, 2003). In the cooperative scheme any system has information from the other systems, whereas in the coordinated scheme there exists a leader which dictates the behavior to the rest of the systems. Both schemes involve synchronization. The cooperative scheme can be seen as a complex network, where there exists an arrangement of nodes or systems which are connected or linked by a coupling force, moreover there exists a connection matrix which determines the topology of the network or the interaction between systems (Albert and Barabasí, 2002; Wang and Chen, 2003; Boccaletti *et al.*, 2002). Coordinated scheme concerns with the master-slave synchronization, where there is a master or leader system and one or many slaves, and any slave receives information from the leader (Femat and Solís-Perales, 2008; Boccaletti *et al.*, 2002).

We consider that four robots are synchronized if they perform the same task at the same time, whereas the synchronization between the articular coordinates is given by the inverse and direct kinematics transformations, thus the robots are synchronous in the work space and in the articular space. The aim of studying synchronization of mechanical systems is that in real processes like manufacturing, biomedical, automotive, it is required more efficiency, quality and precision in the resulting product, therefore, the coordinated and cooperative schemes have been developed.

Parallel mechanical architectures have been originally proposed in the context of tire-testing machines and flight simulators (Stewart, 1965). Parallel kinematic chains have recently attracted attention to machine tool field (Parallel Kinematic Machine-PKM) and automation because of their conceptual potentials in high motion dynamics and accuracy, combined with high structural rigidity due to their closed kinematic loops (Weck and Staimer, 2002). The main motivation behind the use of such architectures is that they provide better stiffness and accuracy than serial kinematic chains. Moreover, they allow the actuators to be fixed to the base –or to be located close to the base– of the mechanism, which minimizes the inertia of the moving parts and makes possible to use more powerful actuators (Gosselin *et al.*, 1996). Because the external load can be shared by the actuators, parallel manipulators tend to have a large load-carry capacity. However, they suffer the problems or relatively small useful workspace and design difficulties (Tsai, 1999).

The main idea of the present work is to show that closed kinematic chains can be synchronized when they are connected in a network form. We use a classical control action provided by a Proportional and Derivative controller which are very common in robotics, however this controller requires some information in order to tracks the trajectory, nevertheless, we consider that this information

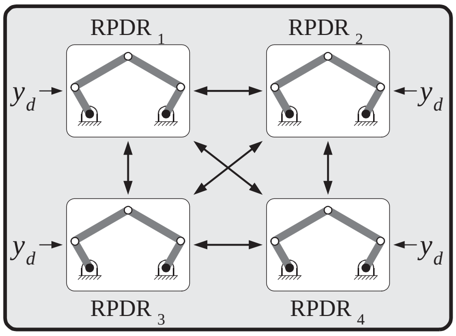


Fig. 1. Robot essemble of four RPDR robots.

is available for feedback. Another controller can provide the same result, in other words, the synchronization of strictly different robots could be obtained independently of the control strategy.

This paper is organized as follows, the section II presents the problem statement, in sections III and IV the mechanical system description is provided, the simulation results are presented in section V and finally the work is closed with some conclusions in section VI.

II. PROBLEM STATEMENT

The mutual synchronization problem can be seen as a network of robots linked by a coupling force, moreover, each robot has connected a controller and is provided by the reference signal. There are two classes of networks, small world networks are characterized by posses a small relative average distance between nodes and scale free networks where a small number of nodes are highly connected and the rest of the nodes are connected to some of this nodes. Therefore we consider a small world network of robots. As was stated in the Introduction, the synchronization of robots is considered when the systems track the same trajectory at the same time, this is, the trajectory in the workspace is tracked by each robot.

The dynamics of a mechanical system considered is such that in absence of friction and other disturbances is obtained using Lagrangian dynamics and is given by the following set of ordinary differential equations

$$\mathbf{D}'(\mathbf{q})\ddot{\mathbf{q}} + \mathbf{C}'(\mathbf{q},\dot{\mathbf{q}})(\dot{\mathbf{q}}) + \mathbf{G}'(\mathbf{q}) = \boldsymbol{\tau} \qquad (1)$$

where $\mathbf{q}(t) \in \mathbb{R}^n$ is the vector of joint variables; $\mathbf{D}'(\mathbf{q}) \in \mathbb{R}^{n\times n}$ is the inertia matrix; $\mathbf{C}'(\mathbf{q},\dot{\mathbf{q}}) \in \mathbb{R}^{n\times n}$ represents the Coriolis and centrifugal forces, $\mathbf{G}'(\mathbf{q}) \in \mathbb{R}^n$ is the vector of gravitational forces, and the control input torque is $\boldsymbol{\tau} \in \mathbb{R}^n$. The inertia matrix $\mathbf{D}'(\mathbf{q})$ is symmetric, positive definite and is uniformly bounded i.e.,

$$m_1\|\mathbf{x}\|^2 \le \mathbf{x}^T\mathbf{D}'(\mathbf{q})\mathbf{x} \le m_2\|\mathbf{x}\|^2, \quad \forall \mathbf{x} \in \Re^n \qquad (2)$$

where it is assumed that m_1 and m_2 are known positive constants that depend on the mass properties of the specific robot manipulator. The matrix $\dot{\mathbf{D}}'(\mathbf{q}) - 2\mathbf{C}'(\mathbf{q},\dot{\mathbf{q}})$ is skew-symmetric.

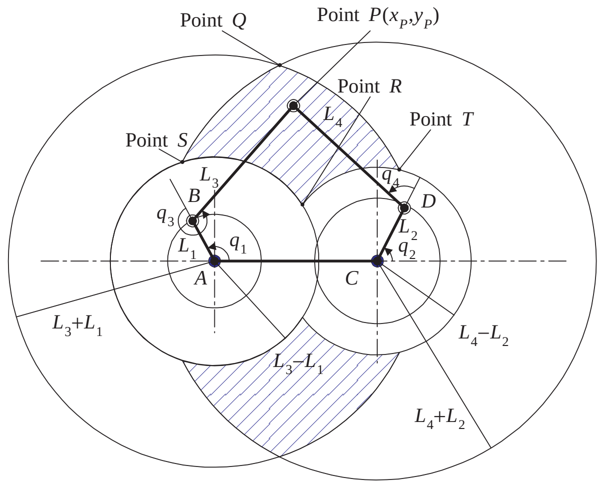


Fig. 2. Planar five-bar parallel robot and its workspace.

Once the trajectory is defined, we propose a torque computed PD controller and also consider the interaction between robots. It is given as follows

$$\begin{aligned}\tau_i = {} & \mathbf{D}_i(q_i)\ddot{q}_{ri} + \mathbf{C}_i(q_i,\dot{q}_i)\dot{q}_{ri} + \mathbf{G}_i(q_i) \\ & - K_{p,i}e_i - K_{d,i}\dot{e}_i\end{aligned} \qquad (3)$$

for $i = 1, 2, ..., N$ and the error term $e_i = q_i - q_{ri}$ includes the diffusible connection of the network and it is given by

$$e_i = q_i - q_{ri} = q_i - \{q_d - c\sum_{j=1}^{N} a_{ij}q_j\} \qquad (4)$$

where q_d is the desired trajectory provided by the signal to be tracked, q_{ri} is the deviation of the robot trajectories from the desired signal which includes the effects of the robots in the network, $a_{i,j}$ defines the network topology, in other words the interconnection between the robots, c is the coupling strength which links each robot, $K_{p,i}$, $K_{d,i}$ and $K_{I,i}$ are positive definite gain matrices which can be determined in such manner that the closed-loop dynamics be stable (Nijmeijer and Rodriguez-Angeles, 2003).

III. KINEMATIC PROBLEMS

In Fig. 2 we can see that the end effector point $P(x_P, y_P)$, points up and Q, R, S, T define the workspace where the desired trajectory is implemented to perform synchronization of n robots, each with similar architecture.

In this part is developed the direct and inverse kinematics using a similar methology presented in (Liu *et al.*, 2006)

III-A. *Inverse Kinematics*

The inverse kinematic problem can be solved by writting following constraint equations

$$|PB| = L_3 \quad |PD| = L_4 \qquad (5)$$

in another form

$$[x - L_1\cos(q_1)]^2 + [y - L_1\sin(q_1)]^2 = L_3^2 \qquad (6)$$

$$[x - L_5 - L_2\cos(q_2)]^2 + [y - L_2\sin(q_2)]^2 = L_4^2 \quad (7)$$

from above equations, if the position of P is known the variables q_1 and q_2 can be obtained as

$$q_i = 2\tan^{-1}(z_i)\,, \quad i = 1, 2$$

where

$$z_i = \frac{-b_i + \sigma_i\sqrt{b_i^2 - 4a_ic_i}}{2a_i} \quad (8)$$

in which

$$\begin{aligned}
\sigma_i &= 1 \quad \text{or} \quad -1\\
a_1 &= x^2 + y^2 + L_1^2 - L_3^2 + 2xL_1\\
b_1 &= -4yl_1\\
c_1 &= x^2 + y^2 + L_1^2 - L_3^2 - 2xL_1\\
a_2 &= x^2 + y^2 + L_2^2 - L_4^2 + L_5^2 - 2xL_5 - 2L_2(L_5 - x)\\
b_2 &= -4yl_2\\
c_2 &= x^2 + y^2 + L_2^2 - L_4^2 + L_5^2 - 2xL_5 + 2L_2(L_5 - x)
\end{aligned}$$

For the configuration shown in Fig. 2 is necessary that $\sigma_1 = 1$ and $\sigma_2 = -1$.

III-B. *Forward Kinematics*

The position of output point $P(x, y)$ with respect to input angles q_1 and q_2 is determined from the following equations

$$x^2 + y^2 - 2[L_1\cos(q_1)]x - 2[L_1\sin(q_1)]y + L_1^2 - L_3^2 = 0 \quad (9)$$

$$x^2 + y^2 + 2x[L_2\cos(q_2) - L_5] - 2y[L_2\sin(q_2)] + 2L_2L_5\cos(q_2) + L_5^2 + L_2^2 - L_4^2 = 0 \quad (10)$$

From (9)–(10) yields

$$x = ey + f \quad (11)$$

where

$$\begin{aligned}
e &= \frac{L_2\sin(q_2) - L_1\sin(q_1)}{-L_2\cos(q_2) - L_5 + l_1\cos(q_1)}\\
f &= \frac{L_1^2 - L_2^2 - L_3^2 + L_4^2 - L_5^2 - 2L_2L_5\cos(q_2)}{2(-L_2\cos(q_2) - L_5 + L_1\cos(q_1))}
\end{aligned}$$

Substituting (11) to (9) yields

$$dy^2 + gy + h = 0 \quad (12)$$

in which

$$\begin{aligned}
d &= 1 + e^2\\
g &= 2(ef - L_1e\cos(q_1) - L_1\sin(q_1))\\
h &= f^2 - 2fL_1\cos(q_1) + L_1^2 - L_3^2
\end{aligned}$$

so, y can be obtained as

$$y = \frac{-g + \sigma\sqrt{g^2 - 4dh}}{2d} \quad (13)$$

where $\sigma = 1$ corresponds to the configuration shown in Fig. 2.

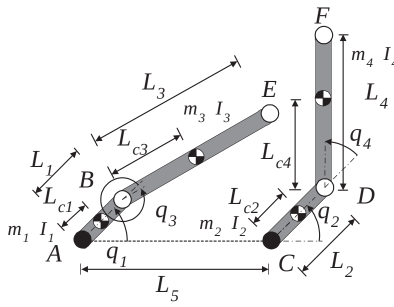


Fig. 3. Free system.

IV. Reduced Dynamic Model

The reduced model method of closed chain mechanisms presented in (Ghorbel *et al.*, 2000) is employed to obtain the dynamic model of the five-bar parallel robot.

$$D(q')\ddot{q} + C(q', \dot{q}')\dot{q} + G(q') = \boldsymbol{\tau} \quad (14)$$

$$\dot{q}' = \rho(q')\dot{q} \quad (15)$$

$$q' = \sigma(q) \quad (16)$$

$$D(q') = \rho(q')^T D'(q')\rho(q') \quad (17)$$

$$C(q', \dot{q}') = \rho(q')^T C'(q', \dot{q}')\rho(q') + \rho(q')^T D'(q')\dot{\rho}(q', \dot{q}') \quad (18)$$

$$G(q') = \rho(q')^T G'(q') \quad (19)$$

The first step in deriving the equations of motion is the selection of free system. In our free system, the robot is virtually cut open in the end-effector, resulting in two serial robots each with two dofs as shown in Fig. 3 (Ghorbel *et al.*, 2000). In this, m_i, L_i, L_{ci}, are respectively, the mass, length of link i, and distance to the center of mass. The inertia of link i about the line through the center of mass parallel to the axis of rotation is denoted by I_i. Thus the constraint equations are due to point E being coincident with point F and are given by

$$\phi(q') = \begin{bmatrix} L_1\cos(q_1) + L_3\cos(q_1 + q_3) - L_5 - L_2\cos(q_2) - L_4\cos(q_2 + q_4) \\ L_1\sin(q_1) + L_3\sin(q_1 + q_3) - L_2\sin(q_2) - L_4\sin(q_2 + q_4) \end{bmatrix} \quad (20)$$

where $q' = [q_1\ q_2\ q_3\ q_4]^T$ is the generalized coordinate vector of the free system and $q = [q_1\ q_2]^T \in \mathbb{R}^2$ is the independent generalized coordinate vector of position of the actuated links. Since the joints q_1 and q_2 are actuated, we choose the vector of generalized coordinates of the constrained system as $\mathbf{q} = [q_1\ q_2]^T$. The parameterization $\alpha(q') = q$ is given by

$$\alpha(q') = \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \end{bmatrix} q' = q \quad (21)$$

Define

$$\psi(q') \triangleq \begin{bmatrix} \phi(q') \\ \alpha(q') \end{bmatrix}, \quad \psi_{q'}(q') \triangleq \frac{\partial\psi}{\partial q'} \quad (22)$$

obtaining

$$\psi_{q'}(q') = \begin{bmatrix} \psi_{q'}(1,1) & \psi_{q'}(1,2) & \psi_{q'}(1,3) & \psi_{q'}(1,4) \\ \psi_{q'}(2,1) & \psi_{q'}(2,2) & \psi_{q'}(2,3) & \psi_{q'}(2,4) \\ 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \end{bmatrix} \quad (23)$$

where

$$\psi_{q'}(1,1) = -L_1 \sin(q_1) - L_3 \sin(q_1 + q_3)$$
$$\psi_{q'}(1,2) = L_2 \sin(q_2) + L_4 \sin(q_2 + q_4)$$
$$\psi_{q'}(1,3) = -L_3 \sin(q_1 + q_3)$$
$$\psi_{q'}(1,4) = L_4 \sin(q_2 + q_4)$$
$$\psi_{q'}(2,1) = L_1 \cos(q_1) + L_3 \cos(q_1 + q_3)$$
$$\psi_{q'}(2,2) = -L_2 \cos(q_2) - L_4 \cos(q_2 + q_4)$$
$$\psi_{q'}(2,3) = L_3 \cos(q_1 + q_3)$$
$$\psi_{q'}(2,4) = -L_4 \cos(q_2 + q_4)$$

$\rho(q')$ can be expressed as follows:

$$\rho(q') = \psi_{q'}^{-1}(q') \begin{bmatrix} 0 & 0 \\ 0 & 0 \\ 1 & 0 \\ 0 & 1 \end{bmatrix} \quad (24)$$

and

$$\dot{\rho}(q', \dot{q}') = -\psi_{q'}^{-1}(q')\dot{\psi}_{q'}(q', \dot{q}')\rho(q') \quad (25)$$

$D'(q')$, $C'(q', \dot{q}')$, $G'(q')$, are determined as follows. By means of Lagrangian methods (Tsai, 1999) we can obtain the inertia matrix $D'(q') \in \mathbb{R}^{4\times4}$:

$$D'(q') = \sum_{i=1}^{n} \left(J_{vi}^T m_i J_{vi} + J_{\omega i}^T I_i J_{\omega i} \right) \quad (26)$$

$$D'(q') = \begin{bmatrix} d_{1,1} & 0 & d_{1,3} & 0 \\ 0 & d_{2,2} & 0 & d_{2,4} \\ d_{3,1} & 0 & d_{3,3} & 0 \\ 0 & d_{4,2} & 0 & d_{4,4} \end{bmatrix} \quad (27)$$

where

$$d_{1,1} = m_1 L_{c1}^2 + m_3 \left(L_1^2 + L_{c3}^2 + 2L_1 L_{c3} \cos(q_3) \right) + I_1 + I_3$$
$$d_{1,3} = m_3 \left(L_{c3}^2 + L_1 L_{c3} \cos(q_3) \right) + I_3$$
$$d_{2,2} = m_2 L_{c2}^2 + m_4 \left(L_2^2 + L_{c4}^2 + 2L_2 L_{c4} \cos(q_4) \right) + I_2 + I_4$$
$$d_{2,4} = m_4 \left(L_{c4}^2 + L_2 L_{c4} \cos(q_4) \right) + I_4$$
$$d_{3,1} = d_{1,3}$$
$$d_{3,3} = m_3 L_{c3}^2 + I_3$$
$$d_{4,2} = d_{2,4}$$
$$d_{4,4} = m_4 L_{c4}^2 + I_4$$

TABLE I

PARAMETERS OF THE MECHANICAL SYSTEM

Link	m_i (kg)	L_i (m)	L_{ci} (m)	I_i (kg · m^2)
1	0.91	0.08	0.006	0.000847
2	0.28	0.10	0.028	0.000630
3	0.38	0.25	0.125	0.004002
4	0.38	0.25	0.125	0.004002
5	–	0.25	–	–

TABLE II

INITIAL POSITIONS FOR EACH COORDINATE OF THE RPDR'S

RPDR	$q_1(0)$	$q_2(0)$
1	0.00	0.00
2	1.5π	1.2π
3	π	π
4	$\pi/4$	$\pi/4$

Coriolis matrix is:

$$C'(q', \dot{q}') = \begin{bmatrix} c_1 \dot{q}_3 & 0 & c_1(\dot{q}_1 + \dot{q}_3) & 0 \\ 0 & c_2 \dot{q}_4 & 0 & c_2(\dot{q}_2 + \dot{q}_4) \\ -c_1 \dot{q}_1 & 0 & 0 & 0 \\ 0 & -c_2 \dot{q}_2 & 0 & 0 \end{bmatrix} \quad (28)$$

where $c_1 = -m_3 L_1 L_{c3} \sin(q_3)$, and $c_2 = -m_4 L_2 L_{c4} \sin(q_4)$. Gravity vector is

$$G'(q') = g \begin{bmatrix} (m_1 L_{c1} + m_3 L_1) \cos(q_1) + m_3 L_{c3} \cos(q_1 + q_3) \\ (m_2 L_{c2} + m_4 L_2) \cos(q_2) + m_4 L_{c4} \cos(q_2 + q_4) \\ m_3 L_{c3} \cos(q_1 + q_3) \\ m_4 L_{c4} \cos(q_2 + q_4) \end{bmatrix} \quad (29)$$

where $g = 9.81$ m/s^2 is the gravitational acceleration constant.

$$\psi(q') \triangleq \begin{bmatrix} \phi(q') \\ \alpha(q') \end{bmatrix} \quad (30)$$

Manipulating (20) by a similar procedure to obtain the well known Freudenstein's equation, we can get q_4 and q_3 as

$$q_4 = 2\tan^{-1}\left[\frac{B + \sigma_i \sqrt{A^2 + B^2 - C^2}}{A + C} \right] - q_2 \quad (31)$$

$$q_3 = \tan^{-1}\left[\frac{\mu + L_4 \sin(q_2 + q_4)}{\lambda + L_4 \cos(q_2 + q_4)} \right] - q_1 \quad (32)$$

where $\lambda = -L_1 \cos(q_1) + L_2 \cos(q_2) + L_5$, $\mu = -L_1 \sin(q_1) + L_2 \sin(q_2)$, $A = 2L_4\lambda$, $B = 2L_4\mu$, and $C = L_3^2 - L_4^2 - \lambda^2 - \mu^2$.

V. SIMULATION RESULTS

We have four robots with identical configurations and properties (see Table I). Each one of the robots has a model based control, i.e., computed-torque control PD (neither disturbances nor friction are considered)(Lewis *et al.*, 2003). All robots have implemented the same types of control and with identical gains. They differ only in their inicial position, as indicated in Table II. The gains of the controllers are $K_p = 100$ and $K_d = 20$, obtained heuristically. The derivatives of q_4 and q_3 were obtained analytically to avoid the use of numerical derivation. The system was simulated in MATLAB/SIMULINK with R2009b version 7.9. For synchronization was used the mutual synchronization equation

$$y_{\mathrm{ref}k,1}(t) = y_{\mathrm{d}k}(t)$$
$$y_{\mathrm{ref}k,i}(t) = y_{\mathrm{d}k,i-1}(t)$$

where $k = 1, \dots, m$ and $i = 2, \dots, l$. Within each of simulation block the equation above was implemented. The coupling constant factor of each robot is 1. The result of this equation is the reference synchronization that will take control of each robot. Since this equation depends on the interaction of the robots, at one point the four robots come to perform the same action. It is when is said they are already synchronized. Note that not always going to follow the desired reference, i.e., the robots will be doing the same. The desired trajectory for the point P is a circunference that has a radius of 0.053 m and its center is located at $(0.14, 0.21)$ This path has a velocity profile given by the following equation:

$$\theta = 2\pi(10T^3 - 15T^4 + 6T^5) \tag{33}$$

where $T = t/t_{final}$, that in this case $t_{final} = 4$ s. This velocity profile was used to generate the circle

$$x = x_0 + r\cos(\theta) \quad y = y_0 + r\sin(\theta) \tag{34}$$

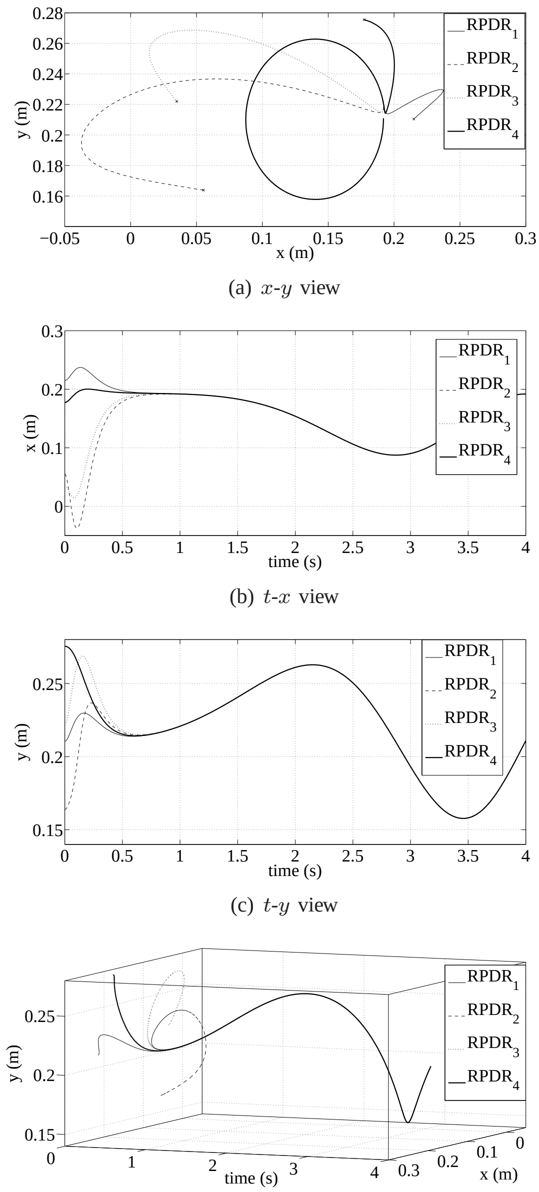
Eq. (34) will generate the coordinates for the Cartesian plane. Using the inverse kinematics we calculate this position in joint coordinates and q_1 and q_2. As method of integration a fixed step ode3 was chosen, with value 0.01 s.

Fig. 4 presents the tracking path of each robot; Fig. 5 and 6 show tracking errors and applied torques, respectively.

VI. Conclusions

Open-chain mechanisms possess some inherent disadvantages, for example, the position accuracy at the endpoint of the long robot arm is considerably low; a small amount of error at each revolution joint is magnified at the endpoint of the arm as its length gets longer; most importantly, the mechanical stiffness of the open-chain construction is inherently poor. As a result, the accuracy of the motion tracking performance can be deteriorated. The research trend in modern machinery development therefore shifts toward the design of a new generation of mechanism, i.e., the closed-chain mechanisms for the position and the trajectory tracking purpose.

In this work we show that there is possible to synchronize closed kinematic chain robots with complex dynamics. Also, the synchronization speed is good enough for our purposes with very short response times. The output torque requires low energy this will allow to use compact actuators. This work sets a precedent for developing adaptive techniques to improve the model of the robot without having to know its exact dynamics a priori, achieving a better level of performance. Also, for further research, complex parallel structures with higher degrees of freedom would be modelled and controlled.



(a) x-y view

(b) t-x view

(c) t-y view

(d) Perspective view

Fig. 4. Tracking of the desired reference trayectory for each RPDR.

References

Afraimovich, V., N.N. Verichev and M.I. Rabinovich (1986). Stochastic synchronization of oscillations in dissipative systems. *Radiofizika* **29**, 1050–1060.

Albert, R. and A.L. Barabasí (2002). Statistical mechanics of complex networks. *Rev. Mod. Phys.* **74**, 47–97.

Boccaletti, S., J. Kurths, G. Osipov, D. L. Valladares and C. S. Zhou (2002). The synchronization of chaotic systems. *Phys. Repts.*

Femat, R. and G. Solís-Perales (2008). *Robust Synchronization of Chaotic Systems via Feedback*. Vol. 278. Springer.

Ghorbel, Fathi H., Olinier Chételat, Ruvinda Gunawardana and Roland Longchamp (2000). Modeling and set point control of closed-chain mechanisms: Theory and experiment. *IEEE Transactions on Control Systems Technology* **8**(5), 801–815.

Gosselin, C.M., S. Lemieux and J. P. Merlet (1996). A new architecture of planar three-degree-of-freedom parallel manipulator. *Proc. of the*

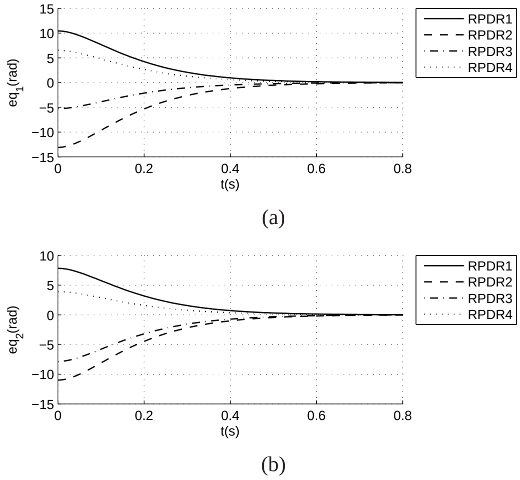


Fig. 5. Tracking errors of the synchronization path.

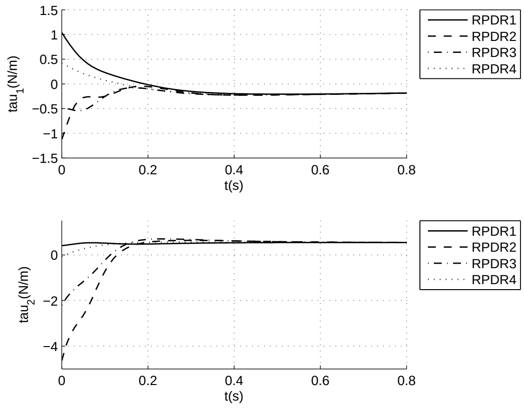


Fig. 6. Applied torque to each RPDR.

International Conference on Robotics and Automation pp. 3738–3743.

Lewis, F.L., D.M. Dawson and C.T. Abdallah (2003). *Robot Manipulator Control: Theory and Practice*. 2 ed. CRC Press.

Liu, Xin-Jun, Jinsong Wang and G. P. Pritschow (2006). Kinematics, singularity and workspace of planar 5r symmetrical parallel mechanisms. *Mechanism and Machine Theory* **41**(2), 145–169.

Nijmeijer, H. and A. Rodriguez-Angeles (2003). Synchronization of mechanical systems. *World Scientific Seriers on Nonlinear Science*. Series A.

Pikovsky, A., M. Rosenblum and J. Kurths. (2008). *Synchronization: A Universal Concept in Nonlinear Sciences*. Cambridge University Press.

Stewart, D. (1965). A platform with six degrees of freedom. *Proc. of the Inst. Of Mech. Eng.* **180**(5), 371–378.

Tsai, L. W. (1999). *Robot Analysis: The Mechanics of Serial and Parallel Manipulators*. John Wiley & Sons.

Wang, X.F. and G. Chen (2003). Complex networks: Small-world, scale-free and beyond. *IEEE Circs. and Sys. Magazine* pp. 6–20.

Weck, M. and D. Staimer (2002). Parallel kinematic machine tools - current state and future potentials. *CIRP Annals - Manufacturaing Technology* **51**(2), 671–683.